Pt. Ravishankar Shukla University, Raipur

Scheme of Examination

M.A./M.Sc. Mathematics (Previous) (Code – 303)

2010-11 & Onwards

There shall be five papers in M.A./ M.Sc. (Previous) Mathematics. All are compulsory. Each theory paper (Paper I – Paper V) will have 100 Marks and divided into five units. However, there will be internal choice in each Unit. **Overall tally of marks will be 500.**

Paper	Description	**Theory**	**Practical**	**Remark**
I	Advanced Abstract Algebra (Code 101)	100	-	-
II	Real Analysis (Code 102)	100	-	-
III	Topology (Code 103)	100	-	-
IV	Complex Analysis (Code 104)	100	-	-
V	Advanced Discrete Mathematics (Code 105)	100	-	-

DETAILS OF SYLLABUS
PAPER –I (Paper code-0961)
Advanced Abstract Algebra

Unit-I Groups - Normal and Subnormal series. Composition series. Jordan-Holder theorem. Solvable groups. Nilpotent groups.

Field theory- Extension fields. Algebraic and transcendental extensions. Separable and inseparable extensions. Normal extensions. Perfect fields. Finite fields. Primitive elements. Algebraically closed fields.

Unit-II Automorphisms of extensions. Galois extensions. Fundamental theorem of Galois theory. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

Unit-III Modules - Cyclic modules. Simple modules. Semi-simple modules. Schuler's Lemma. Free modules. Noetherian and artinian modules and rings-Hilbert basis theorem. Wedderburn Artin theorem. Uniform modules, primary modules, and Noether-Lasker theorem.

Unit-IV Linear transformations- Algebra of linear transformations, characteristic roots,Matrices of linear transformations.

Canonical Forms - Similarity of linear transformations . Invariant subspaces. Reduction to triangular forms. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation. The primary decomposition theorem. Jordan blocks and Jordan forms.

Unit-V Smith normal form over a principal ideal domain and rank. Fundamental structure theorem for finitely generated modules over a Principal ideal domain and its applications to finitely generated abelian groups. Rational canonical from. Generalized Jordan form over any field.

Books Recommended:

1. P.B.Bhattacharya, S.K.Jain, S.R.Nagpaul : Basic Abstract Algebra, Cambridge University press
2. I.N.Herstein : Topics in Albegra, Wiley Eastern Ltd.
3. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.

References

1. M.Artin, Algeabra, Prentice -Hall of India, 1991.
2. P.M. Cohn, Algebra,Vols. I,II &III, John Wiley & Sons, 1982,1989,1991.
3. N.Jacobson, Basic Algebra, Vols. I , W.H. Freeman, 1980 (also published by Hindustan Publishing Company).
4. S.Lang, Algebra, 3rd edition, Addison-Wesley, 1993.
5. I.S. Luther and I.B.S. Passi, Algebra, Vol. I-Groups, Vol.II-Rings, Narosa Publishing House (Vol.l-1996,Vol. II-1999)

6. D.S.Malik, J.N.Mordeson, and M.K.Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition,1997.
7. Quazi Zameeruddin and Surjeet Singh : Modern Algebra
8. I. Stewart, Galois theory, 2nd edition, chapman and Hall, 1989.
9. J.P. Escofier, Galois theory, GTM Vol.204, Springer, 2001..
10. Fraleigh , A first course in Algebra Algebra, Narosa,1982.
11. K.B. Datta, Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi,2000.
12. S.K.jain,A. Gunawardena and P.B Bhattacharya, Basic Linear Algebra with MATLAB, Key College Publishing (Springer-Verlag),2001.
13. S.Kumaresan, Linear Algebra, A Geometric Approach, Prentice-Hall of India, 2000.
14. T.Y. Lam, lectures on Modules and Rings, GTM Vol. 189, Springer-Verlag,1999.
15. D.S. Passman, A Course in Ring Theory, Wadsworth and Brooks/Cole Advanced Books and Softwares, Pacific groves. California, 1991.

PAPER- II (Paper code-0962)
Real Analysis

Unit-I Definition and existence of Riemann-Stieltjes integral, Properties of the Integral, integration and differentiation, the fundamental theorem of Calculus, integration of vector-valued functions, Rectifiable curves.

Unit-II Rearrangement of terms of a series, Riemann's theorem. Sequences and series of functions, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weierstrass approximation theorem. Power series, uniqueness theorem for power series, Abel's and Tauber's theorems.

Unit-III Functions of several variables, linear transformations, Derivatives in an open subset of R^n, Chain rule, Partial derivatives, interchange of the order of differentiation, Derivatives of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem. Jacobians, extremum problems with constraints, Lagrange's multiplier method, Differentiation of integrals. Partitions of unity, Differential forms, Stoke's theorem.

Unit-IV Lebesgue outer measure. Measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability. Non-measurable sets. Integration of Non-negative functions. The General integral. Integration of Series. Reimann and Lebesgue Integrals. The Four derivatives. Functions of bounded variations.Lebesgue Differentiation Theorem. Differentiation and Integration.

Unit-V Measures and outer measures, Extension of a measure. Uniqueness of Extension. Completion of a measure. Measure spaces. Integration with respect to a measure. The L^p-spaces. Convex functions. Jensen's inequality. Holder and Minkowski inequalities. Completeness of L^p, Convergence in Measure, Almost uniform convergence.

Recommended Books:

1. Principle of Mathematical Analysis By Walter Rudin(3rd edition) McGraw-Hill, 1976, International student edition.
2. Real Analysis By H.L.Roydon, Macmillan Pub.Co.Inc.4th Edition, New York .1962.

References

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi,1985.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar,Inc. New York,1975.
3. A.J. White, Real Analysis; an introduction, Addison-Wesley Publishing Co.,Inc.,1968.
4. G.de Barra, Measure Theory and Integration, Wiley Eastern Limited, 1981.

5. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
6. P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited Published, New Delhi, 1986 Reprint 2000).
7. I.P. Natanson, Theory of Functions of a Real Variable. Vol. l, Frederick Ungar Publishing Co., 1961.
8. Richard L. Wheeden and Antoni Zygmund, Measure and Integral: An Introduction to Real Analysis, Marcel Dekker Inc.1977.
9. J.H. Williamson, Lebesgue Integration, Holt Rinehart and Winston, Inc. New York. 1962.
10. A. Friedman, Foundations of Modern Analysis, Holt, Rinehart and Winston, Inc., New York, 1970.
11. P.R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950.
12. T.G. Hawkins, Lebesgue's Theory, of Integration: Its Origins and Development, Chelsea, New York, 1979.
13. K.R. Parthasarathy, Introduction to Probability and Measure, Macmillan Company of India Ltd., Delhi, 1977.
14. R.G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966.
15. Serge Lang, Analysis I & II, Addison-Wesley Publishing Company, Inc. 1969.
16. Inder K. Rana, An Introduction to Measure and Integration, Norosa Publishing House, Delhi, 1997.
17. Walter Rudin, Real & Complex Analysis, Tata McGraw-Hill Publishing Co.Ltd. New Delhi, 1966.

PAPER-III (Paper code-0963)
Topology

Unit-I Countable and uncountable sets. Infinite sets and the Axiom of Choice. Cardinal numbers and its arithmetic. Schroeder-Bernstein theorem. Cantor's theorem and the continuum hypothesis. Zorn's lemma, well-ordering theorem. Definition and examples of topological spaces. Closed sets. Closure. Dense subsets. Neighbourhoods. Interior, exterior and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology.
Alternate methods of defining a topology in terms of terms of Kuratowski Closure Operator and Neighbourhood Systems.

Unit-II Continuous functions and homeomorphism. First and Second Countable spaces. Lindelof's theorems. Separable spaces. Second countability and separability.Separation axioms T_0, T_1, T_2, $T_{3^1/_2}$,T_4; their Characterizations and basic properties. Urysohn's lemma, Tietze extension theorem.

Unit-III Compactness. Continuous functions and compact sets. Basic properties of Compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Stone-Cech compactification. Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric space. Connected spaces. Connectedness on the real line. Components. Locally connected spaces.

Unit-IV Tychonoff product topology in terms of standard sub-base and its characterizations. Projection maps. Separation axioms and product spaces. Connectedness and product spaces. Compactness and product spaces (Tychonoff's theorem). Countability and product spaces. Embedding and metrization. Embedding lemma and Tychonoff embedding. The Urysohn metrization theorem. Metrization theorems and Paracompactness-Local finiteness. The Nagata-Smirnov metrization theorem. Paracompactness. The Smirnov metrization theorem.

Unit-V The fundamental group and covering spaces-Homotopy of paths. The fundamental group. Covering spaces. The fundamental group of the circle and the fundamental theorem of algebra. Nets and filter. Topology and convergence of nets. Hausdorffness and nets. Compactness and nets. Filters and their convergence. Canonical way of converting nets to filters and vice-versa. Ultra-filters and Compactness.

Recommended Books:

1. Topology, A First Course By James R. Munkres, Prentice Hall of India Pvt. Ltd., New Delhi,2000.
2. Introduction to General Topology By K.D.Joshi, Wiley Eastern Ltd., 1983.

References

1. J. Dugundji, Topology, Allyn and Bacon, 1966 (reprinted in India by Prentice Hall of India Pvt. Ltd.).

2. George F.Simmons, Introduction to Topology and modern Analysis, McGraw-Hill Book Company, 1963.
3. J.Hocking and G Young, Topology, Addison-Wiley Reading, 1961.
4. J.L. Kelley, General Topology, Van Nostrand, Reinhold Co., New York,1995.
5. L. Steen and J. Seebach, Counter examples in Topology, Holt, Rinehart and Winston, New York, 1970.
6. W.Thron, Topologically Structures, Holt, Rinehart and Winston, New York,1966.
7. N. Bourbaki, General Topology Part I (Transl.),Addison Wesley, Reading, 1966.
8. R. Engelking, General Topology, Polish Scientific Publishers, Warszawa, 1977.
9. W. J. Pervin, Foundations of General Topology, Academic Press Inc. New York,1964.
10. E.H.Spanier, Algebraic Topology, McGraw-Hill, New York,1966.
11. S. Willard, General Topology, Addison-Wesley, Reading, 1970.
12. Crump W.Baker, Introduction to Topology, Wm C. Brown Publisher, 1991.
13. Sze-Tsen Hu, Elements of General Topology, Holden-Day,Inc.1965.
14. D. Bushaw, Elements of General Topology, John Wiley & Sons, New York, 1963.
15. M.J. Mansfield, Introduction to Topology, D.Van Nostrand Co. Inc.Princeton,N.J.,1963.
16. B. Mendelson, Introduction to Topology, Allyn & Bacon, Inc., Boston,1962.
17. C. Berge, Topological Spaces, Macmillan Company, New York,1963.
18. S.S. Coirns, Introductory Topology, Ronald Press, New York, 1961.
19. Z.P. Mamuzic, Introduction to General Topology, P. Noordhoff Ltd.,Groningen, 1963.
20. K.K.Jha, Advanced General Topology, Nav Bharat Prakashan, Delhi.

PAPER-IV (Paper code-0964)

Complex Analysis

Unit-I Complex integration, Cauchy-Goursat. Theorem. Cauchy's integral formula. Higher order derivatives. Morera's Theorem. Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra. Taylor's theorem. Maximum modulus principle. Schwarz lemma. Laurent's series. Isolated singularities. Meromorphic functions. The argument principle. Rouche's theorem Inverse function theorem.

Unit-II Residues. Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to arg z, logz and z^a. Bilinear transformations, their properties and classifications. Definitions and examples of Conformal mappings. Spaces of analytic functions. Hurwitz's theorem. Montel's theorem Riemann mapping theorem.

Unit-III Weierstrass' factorisation theorem. Gamma function and its properties. Riemann Zeta function. Riemann's functional equation. Runge's theorem. Mittag-Leffler's theorem. Analytic Continuation. Uniqueness of direct analytic continuation. Uniqueness of analytic continuation along a curve. Power series method of analytic continuation Schwarz Reflection Principle. Monodromy theorem and its consequences. Harmonic functions on a disk. Harnack's inequality and theorem. Dirichlet Problem. Green's function.

Unit-IV Canonical products. Jensen's formula. Poisson-Jensen formula. Hadamard's three circles theorem. Order of an entire function. Exponent of Convergence. Borel's theorem. Hadamard's factorization theorem.

Unit-V The range of an analytic function. Bloch's theorem. The Little Picard theorem. Schottky's theorem. Montel Caratheodory and the Great picard theorem. Univalent functions. Bieberbach's conjecture (Statement only) and the "1/4-theorem.

Recommended Books:

1. Complex Analysis By L.V.Ahlfors, McGraw - Hill, 1979.
2. J.B. Conway, Functions of one Complex variable, Springer-Verlag, International student-Edition, Narosa Publishing House,1980.

References

1. H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford 1990.
2. Complex Function Theory By D.Sarason
3. Liang-shin Hahn & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
4. S. Lang, Complex Analysis, Addison Wesley, 1977.
5. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
6. Mark J.Ablowitz and A.S. Fokas, Complex Variables: Introduction and Applications, Cambridge University press, South Asian Edition, 1998.
7. E. Hille, Analytic Function Theory (2 Vols.) Gonn & Co., 1959.

8. W.H.J. Fuchs, Topics in the Theory of Functions of one Complex Variable, D.Van Nostrand Co., 1967.
9. C.Caratheodory, Theory of Functions (2 Vols.) Chelsea Publishing Company, 1964.
10. M.Heins, Complex Function Theory, Academic Press, 1968.
11. Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.
12. S.Saks and A.Zygmund, Analytic Functions, Monografic Matematyczne, 1952.
13. E.C Titchmarsh, The Theory of Functions, Oxford University Press, London.
14. W.A. Veech, A Second Course in Complex Analysis, W.A. Benjamin, 1967.
15. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

PAPER-V (Paper code-0965)
Advanced Discrete Mathematics

Unit-I Formal Logic-Statements. Symbolic Representation and Tautologies. Quantifiers, Predicates and Validity. Propositional Logic. Semigroups & Monoids-Definitions and Examples of Semigroups and monoids (including those pertaining to concatenation operation). Homomorphism of semigroups and monoids. Congruence relation and Quotient Semigroups. Subsemigroup and submonoids. Direct Products. Basic Homomorphism Theorem.

Unit-II Lattices-Lattices as partially ordered sets. Their properties. Lattices as Algebraic Systems. Sublattices, Direct products, and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices. Boolean Algebras-Boolean Algebras as Lattices. Various Boolean Identities. The Switching Algebra example. Subalgebras, Direct Products and Homomorphisms. Join-Irreducible elements, Atoms and Minterms. Boolean Forms and Their Equivalence. Minterm Boolean Forms, Sum of Products Canonical Forms. Minimization of Boolean Functions. Applications of Boolean Algebra to Switching Theory (using AND,OR & NOT gates). The Karnaugh Map Method.

Unit-III Graph Theory-Definition of (Undirected) Graphs, Paths, Circuits, Cycles, & Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar Graphs and their properties. Trees. Euler's Formula for connected planar Graphs. Complete & Complete Bipartite Graphs. Kuratowski's Theorem (statement only) and its use. Spanning Trees, Cut-sets, Fundamental Cut -sets, and Cycle. Minimal Spanning Trees and Kruskal's Algorithm. Matrix Representations of Graphs. Euler's Theorem on the Existence of Eulerian Paths and Circuits. Directed Graphs. In degree and Out degree of a Vertex.
Weighted undirected Graphs. Dijkstra's Algorithm.. strong Connectivity & Warshall's Algorithm. Directed Trees. Search Trees. Tree Traversals.

Unit-IV Introductory Computability Theory-Finite State Machines and their Transition Table Diagrams. Equivalence of finite State Machines. Reduced Machines. Homomorphism. Finite Automata. Acceptors. Non-deterministic Finite Automata and equivalence of its power to that of Deterministic Finite Automata. Moore and mealy Machines. Turing Machine and Partial Recursive Functions.

Unit-V Grammars and Languages-Phrase-Structure Grammars. Rewriting Rules. Derivations. Sentential Forms. Language generated by a Grammar. Regular, Context-Free, and Context Sensitive Grammars and Languages. Regular sets, Regular Expressions and the Pumping Lemma. Kleene's Theorem. Notions of Syntax Analysis, Polish Notations.

Conversion of Infix Expressions to Polish Notations. The Reverse Polish Notation.

Recommended Books:

1. Elements of Discrete Mathematics, C.L.Liu, McGraw-Hill Book Co.
2. Discrete Mathematical Structures with Applications to Computer Science, J.P. Tremblay & R. Manohar, McGraw-Hill Book Co., 1997.

References

1. J.L. Gersting, Mathematical Structures for Computer Science, (3rd edition), Computer Science Press, New York.
2. Seymour Lepschutz, Finite Mathematics (International) edition 1983), McGraw-Hill Book Company, New York.
3. S.Wiitala, Discrete Mathematics-A Unified Approach, McGraw-Hill Book Co.
4. J.E. Hopcroft and J.D Ullman, Introduction to Automata Theory, Languages & Computation, Narosa Publishing House.
5. N. Deo. Graph Theory with Application to Engineering and Computer Sciences. Prentice Hall of India
6. K.L.P.Mishra and N.Chandrashekaran .,Theory of Computer Science PHI(2002)